INTEGRALES TRIGONOMETRICAS

$$\int \operatorname{sen}^{par} x\ dx \Rightarrow \operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$$

$$\int \cos^{par} x\ dx \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \operatorname{sen}^{impar} x\ dx = \int \operatorname{sen}^{par} x \cdot \operatorname{sen} x\ dx \Rightarrow \begin{cases} \operatorname{sen}^2 x = 1-\cos^2 x \\ u = \cos x \\ -du = \operatorname{sen} x\ dx \end{cases}$$

$$\int \cos^{impar} x\ dx = \int \cos^{par} x \cdot \cos x\ dx \Rightarrow \begin{cases} \cos^2 x = 1-\operatorname{sen}^2 x \\ u = \operatorname{sen} x \\ du = \cos x\ dx \end{cases}$$

$$\int \tan^{par} x\ dx = \int \tan^{2n} x \cdot \tan^2 x\ dx = \int \tan^{2n} x \cdot (\sec^2 x - 1)\ dx$$
$$= \underbrace{\int \tan^{2n} x \cdot \sec^2 x\ dx}_{\substack{u=\tan x \\ du=\sec^2 x\ dx}} - \int \tan^{2n} x\ dx$$

$$\int \cot^{par} x\ dx = \int \cot^{2n} x \cdot \cot^2 x\ dx = \int \cot^{2n} x \cdot (\csc^2 x - 1)\ dx$$
$$= \underbrace{\int \cot^{2n} x \cdot \csc^2 x\ dx}_{\substack{u=\cot x \\ du=-\csc^2 x\ dx}} - \int \cot^{2n} x\ dx$$

$$\int \tan^{impar} x\ dx = \int \tan^{2n} x \cdot \tan x\ dx = \underbrace{\int (\sec^2 x - 1)^n \cdot \tan x\ dx}_{\substack{u=\sec x \\ du=\sec x \cdot \tan x\ dx}}$$

$$\int \cot^{impar} x\ dx = \int \cot^{2n} x \cdot \cot x\ dx = \underbrace{\int (\csc^2 x - 1)^n \cdot \cot x\ dx}_{\substack{u=\csc x \\ du=-\csc x \cdot \cot x\ dx}}$$

$$\int \sec^{impar} x\ dx = \int \underbrace{\sec^n x}_{u} \cdot \underbrace{\sec^2 x\ dx}_{dv} \Rightarrow \text{integrar por partes}$$

$$\int \csc^{impar} x\ dx = \int \underbrace{\csc^n x}_{u} \cdot \underbrace{\csc^2 x\ dx}_{dv} \Rightarrow \text{integrar por partes}$$

$$\int \sec^{par} x\ dx = \int \sec^{2n} x \cdot \sec^2 x\ dx = \underbrace{\int (1+\tan^2 x)^n \cdot \sec^2 x\ dx}_{u=\tan x\ \wedge\ du=\sec^2 x\ dx}$$

$$\int \csc^{par} x\ dx = \int \csc^{2n} x \cdot \sec^2 x\ dx = \underbrace{\int (1+\cot^2 x)^n \cdot \csc^2 x\ dx}_{u=\cot x\ \wedge\ du=-\csc^2 x\ dx}$$

$$\int (\operatorname{sen}^{par} x)\cdot(\cos^{impar} x)\ dx \Rightarrow \begin{cases} u = \operatorname{sen} x \\ du = \cos x\ dx \\ \cos^2 x = 1-\operatorname{sen}^2 x \end{cases}$$

$$\int (\operatorname{sen}^{impar} x)\cdot(\cos^{par} x)\ dx \Rightarrow \begin{cases} u = \cos x \\ -du = \operatorname{sen} x\ dx \\ \operatorname{sen}^2 x = 1-\cos^2 x \end{cases}$$

$$\int (\operatorname{sen}^{impar} x)\cdot(\cos^{impar} x)\ dx \Rightarrow \textit{Cualquiera}$$

$$\int (\operatorname{sen}^{par} x)\cdot(\cos^{par} x)\ dx \Rightarrow \begin{cases} \operatorname{sen}^2 x = \dfrac{1-\cos 2x}{2} \\ \cos^2 x = \dfrac{1+\cos 2x}{2} \end{cases}$$

$$\left.\begin{matrix} \int \cot^{impar} x \cdot \csc^{par} x\ dx \\ \int \cot^{par} x \cdot \csc^{par} x\ dx \end{matrix}\right\} \Rightarrow \begin{cases} u = \cot x \\ du = -\csc^2 x\ dx \\ \csc^2 x = \cot^2 x + 1 \end{cases}$$

$$\int \cot^{impar} x \cdot \csc^{impar} x\ dx \Rightarrow \begin{cases} u = \csc x \\ du = -\csc x \cdot \cot x\ dx \end{cases}$$

$$\int \cot^{par} x \cdot \csc^{par} x\ dx \Rightarrow \begin{cases} \cot^2 x = \csc^2 x - 1 \\ \textit{Integrar por partes} \end{cases}$$

alternativo

$$\text{cuando "}\cot^{impar} x\text{"} \Rightarrow \begin{cases} \cot x = \dfrac{\cos x}{\operatorname{sen} x} \\ \csc x = \dfrac{1}{\operatorname{sen} x} \end{cases}$$

$$\int \operatorname{sen} ax \cdot \cos bx\ dx = \frac{1}{2}\int \left[\operatorname{sen}(ax+bx) + \operatorname{sen}(ax-bx)\right]$$

$$\int \operatorname{sen} ax \cdot \operatorname{sen} bx\ dx = \frac{1}{2}\int \left[\cos(ax-bx) - \cos(ax+bx)\right]$$

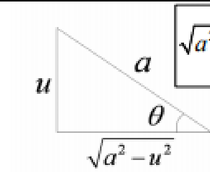
$$\int \cos ax \cdot \cos bx\ dx = \frac{1}{2}\int \left[\operatorname{sen}(ax-bx) + \operatorname{sen}(ax+bx)\right]$$

$$\left.\begin{matrix} \int \tan^{impar} x \cdot \sec^{par} x\ dx \\ \int \tan^{par} x \cdot \sec^{par} x\ dx \end{matrix}\right\} \Rightarrow \begin{cases} u = \tan x \\ du = \sec^2 x\ dx \\ \sec^2 x = \tan^2 x + 1 \end{cases}$$

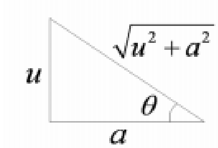
$$\int \tan^{impar} x \cdot \sec^{impar} x\ dx \Rightarrow \begin{cases} u = \sec x \\ du = \sec x \cdot \tan x\ dx \end{cases}$$

$$\int \tan^{par} x \cdot \sec^{impar} x\ dx \Rightarrow \begin{cases} \tan^2 x = \sec^2 x - 1 \\ \textit{Integrar por partes} \end{cases}$$

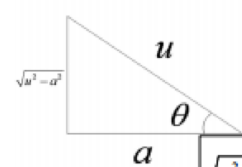
Sustituciones Trigonométricas



$$\sqrt{a^2-u^2} \Rightarrow \begin{cases} \sqrt{a^2-u^2} = a\cos\theta \\ u = a\operatorname{sen}\theta \end{cases}$$



$$\sqrt{a^2+u^2} \Rightarrow \begin{cases} \sqrt{a^2+u^2} = a\sec\theta \\ u = a\tan\theta \end{cases}$$



$$\sqrt{u^2-a^2} \Rightarrow \begin{cases} \sqrt{u^2-a^2} = a\tan\theta \\ u = a\sec\theta \end{cases}$$